Universidad Carlos III de Madrid

Escuela Politécnica Superior

DEPARTAMENTO DE MATEMÁTICAS

First Course. Bachelor's Degree in
Telematics, Communication System and Audiovisual System Engineering

Calculus I. Third Test, December 17th, 2008

GRADE

Surname.. Name..............................
D.N.I. .... Group ....

Time length: 80 min.

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1. Find the following antiderivatives

$$\int \frac{2}{x^2-2x+2}\,dx, \qquad \int \cos^3 x \sin^2 x\,dx \qquad \int x(\log x)^2\,dx$$

[4.5 p.]

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2. Compute the limit

$$\lim_{x\to 0} \frac{\int_0^x e^{t^2}\,dt - x}{x^3}.$$

[1.5 p.]

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3. Find the area enclosed by the graph of the function $f(x) = \dfrac{1}{1+e^{-x}}$, its asymptote at $x \to +\infty$ and the vertical axis. [2 p.]

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4. Compute the volume formed by revolving

$$0 \le y \le 1 + \sin x, \quad 0 \le x \le 2\pi$$

about the x axis: [2 p.]

ANSWERS:

1.

$$\int \frac{2}{x^2-2x+2}\,dx = \int \frac{2}{(x-1)^2+1}\,dx = 2\arctan(x-1)+C.$$

$$\begin{aligned}\int \cos^3 x \sin^2 x\,dx &= \int \cos x(1-\sin^2 x)\sin^2 x\,dx = \int \cos x \sin^2 x\,dx - \int \cos x \sin^4 x\,dx = \\ &= \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C.\end{aligned}$$

For the last one, do IBP twice, first using $u = (\log x)^2$ and second with $u = \log x$,

$$\begin{aligned}\int x(\log x)^2\,dx &= \frac{x^2}{2}\log^2 x - \int 2\frac{\log x}{x}\frac{x^2}{2}\,dx = \\ &= \frac{x^2}{2}\log^2 x - \int x\log x\,dx = \frac{x^2}{2}\log^2 x - \frac{x^2}{2}\log x + \int \frac{x}{2}\,dx = \\ &= \frac{x^2}{2}\log^2 x - \frac{x^2}{2}\log x + \frac{x^2}{4} + C.\end{aligned}$$

2.

$$\lim_{x\to 0}\frac{\int_0^x e^{t^2}\,dt - x}{x^3} \underset{L'H}{=} \lim_{x\to 0}\frac{e^{x^2}-1}{3x^2} \underset{L'H}{=} \lim_{x\to 0}\frac{2xe^{x^2}}{6x} = \frac{1}{3}.$$

3.

$$A = \int_0^\infty \left(1 - \frac{1}{1+e^{-x}}\right)dx = \int_0^\infty \frac{e^{-x}}{1+e^{-x}}dx = -\log\left(1+e^{-x}\right)\Big|_0^\infty = \log 2.$$

4.

$$\begin{aligned}V_x &= \pi\int_0^{2\pi}(1+\sin x)^2 dx = \pi\int_0^{2\pi}(1+2\sin x+\sin^2 x)dx = \\ &= \pi\int_0^{2\pi}\left(1+2\sin x+\frac{1-\cos 2x}{2}\right)dx = \pi\int_0^{2\pi}\left(\frac{3}{2}+2\sin x-\frac{\cos 2x}{2}\right)dx = 3\pi^2+0+0 = 3\pi^2.\end{aligned}$$
